

# Random Interaction Effect of Digital Transformation on General Price Level and Economic Growth

Byung Gwon Choy

Adjunct Professor, bgchoy@snu.ac.kr

Seoul National University, 1 Gwanak-ro, Gwanak-gu, Seoul 08826, Republic of Korea

## Abstract

The paper attempts to evaluate the impact of digital transformation upon productivity using the multi-level structure model of a random interaction effect based on the Bayesian approach to cross-section data. Digital transformation significantly raised general price levels in

Russia and has had consistently significant positive effects upon economic growth through the random interaction effect. Therefore, in Russia in 2018, digital transformation played a role as a driver of technological progress that prompted economic growth rather than economic stability.

**Keywords:** digital transformation; Bayesian theorem; MCMCglmm; random interaction effect

**Citation:** Choy B.G. (2020) Random Interaction Effect of Digital Transformation on General Price Level and Economic Growth. *Foresight and STI Governance*, vol. 14, no 1, pp. 29–47. DOI: 10.17323/2500-2597.2020.1.29.47



© 2020 by the authors. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<http://creativecommons.org/licenses/by/4.0/>).

## Introduction

Digital transformations (DT) have been notable in business fields since 2010. Digital transformation is the intellectual process by which digital technologies are developed (in a similar way to general human development) in all social spheres.

This research suggests that digital transformation can be random and a technical shock, but it is also a phase of technological progress. Thus, at a given point, digital transformation could also be the start of a business cycle and may impact economic growth.

Considering the two-sided nature of digital transformation, this paper first researched what kind of effect it has on productivity, general price level, and economic growth in Russia. Second, this paper distinguishes between the impact of variations in price levels and rates economic growth determined by expert groups. Finally, this study aims to analyze the random interaction effect of digital transformation upon the general price level and economic growth.

## Theoretical Background

To analyze the impact of digital transformation upon the economy, this paper will first consider its impact upon productivity. This is because digital transformation would act as a shock to productivity. This is to determine whether a digital transformation would reduce production costs and improve productivity in Russia in 2018.

Goldfarb et al. [Goldfarb et al., 2015] evaluate the relationship between digitalization and production costs. This author also thinks that digital transformation may reduce operational costs including those related to searches for information and reservation costs. In addition, this paper suggest that digital transformation can reduce production costs including manufacturing, inventory, and management expenses, spending on trade including contract, distribution, and marketing costs. Furthermore, we can take the effect of information costs into consideration. Digital transformation can quickly and easily identify economic risks, thus reducing relevant expenses such as identification costs, moral hazard, and adverse selection. It is expected that deepening the digital transformation and the reduction in overall costs will affect general prices throughout the economy.

Draco et al. [Draco et al., 2015] analyzed ICT's impact upon productivity on the basis of a theorem about the mutual interaction between costs and production. A decrease in the cost of production increases the productivity of a firm because it can produce more output from a given set of production factors. Moreover, this paper hypothesizes that increases in productivity from the digital transformation can directly affect real output on a national scale according to a production function. Thus, digital transformation at

any given time indirectly affects economic growth through changes in productivity.

This paper is an attempt to create four latent variables. Each latent variable has respectively measured variables. The measured variables are the values that were observed during the research survey. Measured variables are selected by on the basis of economic theory. The variables were empirically tested over a long period.

### Measured Variables of Economic Growth (PEG)

Charles I. Jones [Jones, 1995] tested the AK model using time series data. According to the AK growth model, the production function was set as follows:

$$y = Ak, \quad (1)$$

With  $y = Y/L$ ,  $A > 0$  representing the technical level, where,  $y = Y/L$ .  $k = K/L$ .  $Y$ ,  $K$ ,  $L$  respectively represent real output, capital stocks including human capital, and labor productivity.

The digital transformation at any point in time influences the value of  $A$  which represents the technology level in the production function (1). Then the changes in technology level ( $A$ ) directly impact output level from equation (1).

This paper can use this concept as a latent variable, and the latent variable of economic growth (PEG) can be described by seven measured variables described in Table 1. The following can be thought of as the measured variables: the increase in R&D investment, population growth, the intensification of economic activity in networks, the reform of regulations and systems, the increase in the average number of years of education per person, the improvement of productivity, and finally, the increase in investments.

In the study by Caballé and Santos [Caballé, Santos, 1993], human capital and physical capital were determined endogenously and played a major role in determining economic growth. So, this paper uses human capital as one of the measured variables. The average number of years of education per person has been used as a proxy variable for human capital.

As we can see in [Howitt, 1999], there are arguments that population growth may affect the accumulation of human capital. Even if this is not the case, it can be argued that if there is a larger population, there would be a greater number of outstanding members of the workforce. Thus, population growth may determine economic growth. In addition to these variables, other measured variables include social security networks, the reform of regulations and systems, and economic activity networks. In endogenous growth theory, investment in R&D is considered a factor of optimization along with the supply of products on the market. R&D investment is included among the measured variables because it plays an important role in relation to human capital accumulation and innovation policies.

Table 1. Latent Variables and Measured Variables

Latent Variable <sup>1</sup>	Measured Variable	Nature of Measured Variable
Economic growth (PEG)	Increase in R&D investments	Endogenous
	Population growth	Endogenous
	Intensification of economic activity in networks	Endogenous
	Reform of regulations and systems	Endogenous
	Increase in the average number of years of education per person	Endogenous
	Improvement in productivity	Endogenous
	Increase in investments	Endogenous
Digital transformation (DT)	AI	Endogenous
	Mobile banking	Endogenous
	Sharing business	Endogenous
	Fintech	Endogenous
	IoT and smart factory	Endogenous
	Big data and cloud computing	Endogenous
	Navigation applications	Endogenous
	Mobile games	Endogenous
	Autonomous driving cars	Endogenous
Productivity (PRD)	Real wage	Endogenous
	Capital intensity	Endogenous
	Strengthening employee (re-)education	Endogenous
General price level (PRS)	Increase of money supply	Endogenous
	Increase of government expenditure	Endogenous
	Increase of import prices	Endogenous
	Increase of expected inflation rate	Endogenous
	Increase of exchange rate	Endogenous

<sup>1</sup> One out of the four latent variables, Digital transformation(DT), is exogenous. The rest latent variables, Economic growth(PEG), Productivity(PRD) and General price level(PRS), are endogenous.  
Source: compiled by the author.

The above seven measured variables have been introduced as the fundamental factors that determine economic trends in economic growth theory. The measurement variables for economic growth are shown in Table 1.

There has been a long debate over whether an increase in the money supply can affect real national income. Lucas [Lucas, 1972] used a rational expectations theory to prove that money is neutral over the short and long term. In response, Ball and Romer [Ball, Romer, 1990] countered that even if the expectations are rational, the money supply may not be neutral if there is rigidity in the price structure. In this light, we further analyzed whether or not the increase in money supply affected economic growth.

#### **Measured Variables of Digital Transformation(DT)**

Digital transformation products, services, and technologies that are actively used on the market were

selected as measured variables. On the basis of the classification of digital transformation technologies presented in Table 1, we attempted to select the variables for measurement, which adequately characterized the progress of digital transformation in Russia in 2018. The nine measured variables were as follows: (1) AI (Artificial intelligence), (2) Mobile Banking, (3) Sharing Economy, (4) Fintech, (5) IoT (Internet of Things) and Smart Factory, (6) Big Data and Cloud Computing, (7) Navigation Applications, (8) Mobile Games, and (9) Autonomous Self Driving Cars.

#### **Measured Variables of Productivity (PRD)**

In this study the three following measurable variables are used and are sufficient for describing the third latent variable, productivity: real wages, capital intensity ratio, and the training of personnel<sup>1</sup>.

<sup>1</sup> One of reasons why productivity or economic growth is set as a latent variable, even though it can be measured is namely due to Solow's computer paradox. Solow said: «You can see the computer age everywhere except in the productivity statistics.» [Solow, 1987; Triplett, 1999].

First, Akerloff [Akerloff, 1984] presented the efficiency wage hypothesis. Amid asymmetric information, companies can increase their productivity by raising real wages to avoid adverse selection and reduce agent's moral hazard. This paper selected real wages as a measurement variable to account for productivity based on the efficiency wage hypothesis as shown in equation (2).

In the equation,  $y$ ,  $e$ ,  $\omega$  mean real output, worker's work effort, and real wages, respectively.

$$y = f(e(\omega)), f'(\cdot) > 0, e'(\cdot) > 0, \quad (2)$$

Also, this paper selected the capital intensity ratio as the second measured variable. After the production function of Cobb-Douglas was derived, most production functions, such as CES (Constant elasticity of substitution), VES (Variable elasticity of substitution), and a translog function were derived from capital-labor ratio in equation (3). In other words, the capital intensive ratio positively affects worker's average productivity.

In the equation,  $Y/L$ ,  $W/P$ , and  $K/L$  represent average labor productivity, real wage, and the capital intensity ratio, respectively.

$$\ln\left(\frac{Y}{L}\right) = a + b \ln\left(\frac{W}{P}\right) + c \ln\left(\frac{K}{L}\right), \quad (3)$$

where  $b > 0$ ,  $c > 0$

Finally, I used indicators of the level of education of workers and their participation in improving their qualifications and re-education programs. As a result of the accumulation of proficiency, it is possible to obtain a scale-up effect that increases the productivity of each factor of production [Davis et al., 2017].

### Measured Variables of General Price Level (PRS)

The fourth latent variable, general price level, can be measured by monetary growth, fiscal expenditure by the government, imported commodity prices, the foreign exchange rate of the Ruble, and the expected inflation rate in Table 1.

According to the money quantity theory [Friedman, 2017], the long term the growth rate of money is proportional to the inflation rate in equation (4). In the equation,  $M$ ,  $V$ ,  $P$ ,  $T$  stands for money supply, velocity of money circulation, price level, and volume of transaction quantity, respectively. In the equation  $m$ ,  $v$ ,  $\pi$ ,  $t$  stands for the rate of change of  $M$ ,  $V$ ,  $P$ ,  $T$  with respect to time.

$$MV = PT$$

$$m + v = \pi + t,$$

In the long run,  $v = t = 0$

$$\therefore m = \pi. \quad (4)$$

This paper makes an attempt to evaluate the general price level through government expenditure as the measured variable. According to Keynesian theory, if the government has increased fiscal spending, prices on the demand side would fluctuate at least in the long term. There have still been arguments about how much prices will rise when future expectations are introduced, but prices may rise in the middle and long term. This will prompt an increase in the general price level.

This paper also considers the prices of imported goods. If the price of imported goods goes up, it may increase wholesale or retail prices which subsequently pushes overall prices up in a country. Since Russia depends upon overseas imports of daily necessities, rising prices of imported goods are expected to impact Russia's general price level. Import prices are linked to the exchange rate of the Ruble. The exchange rate of the Ruble is being used as a measured variable representing the general price level in Russia.

Finally, this paper uses the expected inflation rate as an evaluation tool. The expected inflation rate was measured taking into account rational expectation theory. The rise in expected prices will raise actual prices in the future. The level of the actual increase depends upon the time horizon (whether short or long term) and upon the type of expectations.

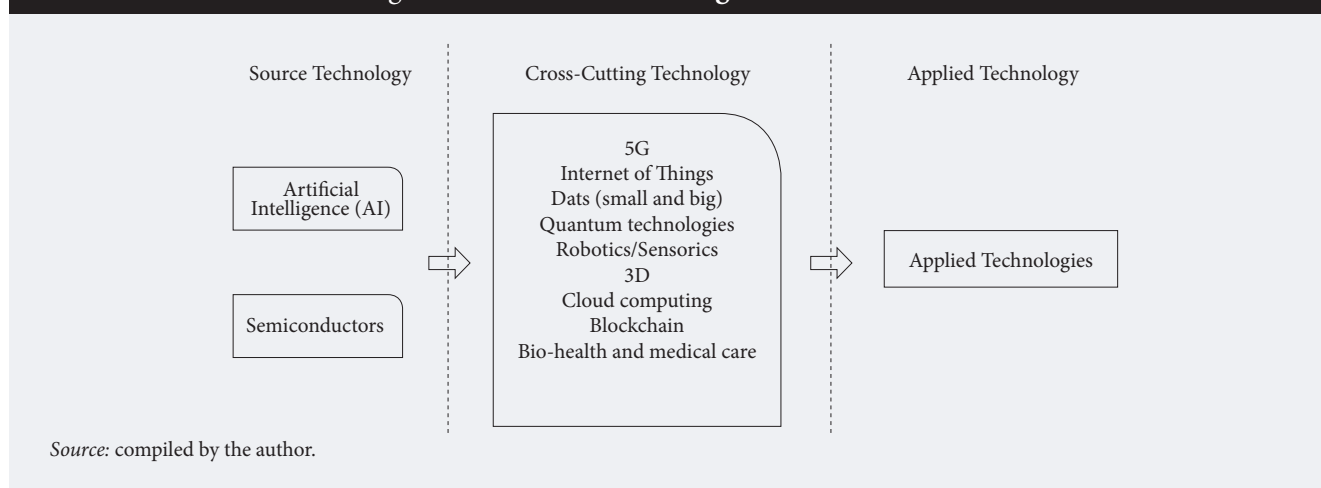
### Qualitative Structure of the Research Survey

To conduct the analysis of digital transformation, the technology of digital transformation, its products, and its services are classified as shown in Figure 1. In Figure 1, digital transformation can be classified as base technologies, cross-cutting technologies, and applied technologies. Source technologies include artificial intelligence (AI) and semiconductors. Applied technology refers to the use of the two base technologies in the real world. Six technologies that have produced a wide variety of application technologies can be categorized as the cross-cutting technologies of digital transformation.

The research survey<sup>2</sup> was conducted face-to-face for about two months in November and December in 2018. The survey participants were a group of experts at the National Research University Higher School Economics (HSE) in Moscow. Respondents were divided into two groups, namely pivotal and non-pivotal. The survey was conducted through a multi-level

<sup>2</sup> The research survey was conducted by providing respondents with simple information according to the rational expectation theory. During the face-to-face survey, if there was a question, the respondent was provided with the necessary information. The questionnaire revolved around residents of the HSE guest house and HSE Moscow. The questionnaire consists of five sections, including digital transformation, productivity, general price level, potential economic growth, and the personal information of the respondents.

Figure 1. Classification of Digital Transformation



model. Experts in each group responded to the four latent and 24 measured variables in Table 1. The collected questionnaire yielded 44 responses. Eight out of 44 surveys were considered pivotal, the other 36 surveys belong to the non-pivotal group. Each individual expert (1st stage) is nested once in the pivotal or non-pivotal group (2nd stage)<sup>3</sup>.

The pivotal group included experts who are able to recommend policies to decision makers in the organization or make policy decisions by themselves. The positions held by those in the pivotal group include directors, deputy directors, members of the editorial committee of the journal, the heads of departments, and the deputy heads. Whether they were in a decision-making unit can be easily verified by the face-to-face surveys. Let us call the pivotal group type1, and the non-pivotal expert group type2.

A unilateral non-parametric Kruskal-Wallis test was conducted to see whether there are any differences between type1 and type2. Although whether one was pivotal was extracted from the survey according to the hierarchy of positions at the organization, this paper tries to confirm whether this distinction is economically and statistically meaningful. The Kruskal-Wallis test was conducted because, as shown in Figure 2, all four latent variables failed to meet the normality. This test was conducted on four latent variables. Those were digital transformation, productivity, general price level, and economic growth, respectively.

In the test, the null and alternative hypotheses are as follows:

H<sub>n</sub>: The distribution of latent variables is the same regardless of the group.

H<sub>a</sub>: At least in one group the distribution of values of the latent variable were distinguished from one another.

A dispersion analysis of the values yielded by the survey responses (independent of the group) was completed where in each of the four latent variables, the normality or equal-variance were considered. In Table 2 there is a statistically significant difference in the productivity variables. The general price level demonstrates a marginally significant difference. These variables rejected the null hypothesis and support the alternative. In addition, there is a difference, although only marginally significant, in the digital transformation variable. Economic growth has been shown to be consistent by supporting the null hypothesis. This analysis means that although the entire sample came from the Higher School of Economics (HSE) in Moscow, there were differences within the group<sup>4</sup>.

## Analytical Model Building

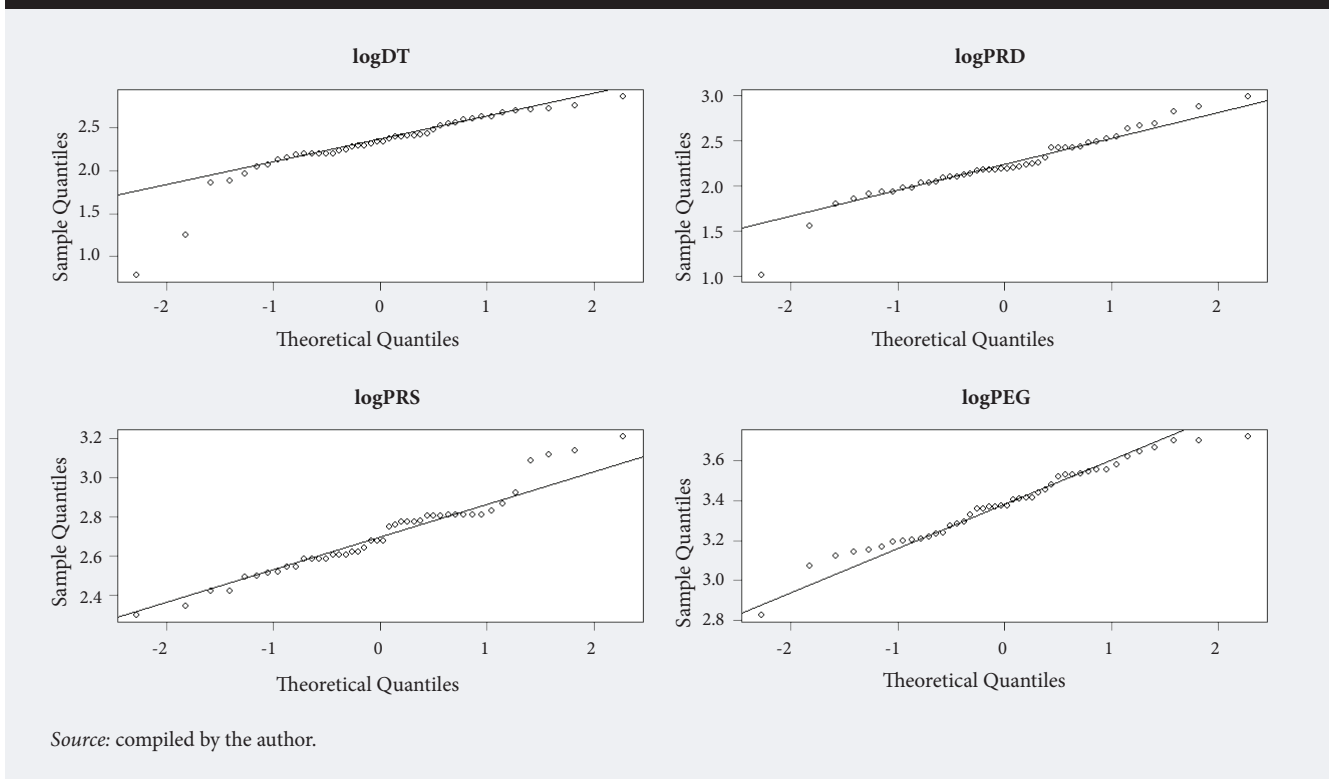
The multi-level response model has two levels. Individual experts were included in either the pivotal group or the non-pivotal group. The model consists of four latent variables: DT, PRD, PEG, and PRS. Here, DT is the external latent variable, while PEG, PRD, and PRS are the internal latent variables that are affected by DT. All internal variables have their internal error respectively. Each latent variable has its respective measured variables. The measured variables are nine DTs, three PRDs, seven PEGs, and five PRSs, respectively as seen in Table 1. All measured variables have measurement errors, there are a total of 24 measurement errors. Thus, the two-level model consists of four latent variables, 24 measured

<sup>3</sup> Moulin [Moulin, 1986] proposed using the key mechanism with quasi-linear utility function to analyze decisions about public goods.

<sup>4</sup> After the pivotal group was also divided into two groups, the unilateral Kruskal-Wallis test was conducted for the three groups in the saturated model.



Figure 2. QQ Normality Test



variables, three internal errors, and 24 measurement errors<sup>5</sup>.

**K Factor Model**

There are several approaches to measuring latent variables [Anderson, Rubin, 1956; Lawley, Maxwell, 1962; Bartholomew et al., 2011]. Joreskog made the Anderson and Rubin approach a statistical application called LISREL8.8 [Joreskog, 1990]. In addition, there are the R2WinBUGS and MCMCglmm instruments for the R program.

Among the several methods for calculating latent variables, this paper constructed a factor analysis model (5)<sup>6</sup>. In this way, it is constructed as follows:

$$Y = ZX + \xi, \text{ where } X \sim N(0, I), \xi \sim N(0, \varphi), \varphi = \text{diag}(\varphi_1, \varphi_2, \dots, \varphi_k). \tag{5}$$

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}, \quad Z = \begin{bmatrix} \rho_{11} & \dots & \rho_{1k} \\ \vdots & \ddots & \vdots \\ \rho_{n1} & \dots & \rho_{nk} \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_k \end{bmatrix}, \quad \xi = \begin{bmatrix} \xi_1 \\ \xi_2 \\ \vdots \\ \xi_n \end{bmatrix}.$$

In the multiple regression analysis equation (5), the measured variable becomes a dependent variable, and the latent variable is an independent variable.

Here the regression coefficient is called factor loading. Factor loading has been used as latent variable value. In this paper, a significant latent variable has factor loading at the level 0.3. To derive these values, we assumed that the residuals were not correlated, and X and  $\xi$  were independent of one another. Every  $X_i$  was assumed to be independent.

**Mixed-Effect Model**

As shown in equation (6), I had to confirm whether the intercept of logDT varies between the type1 and type2 groups. The estimated intercept (1.129) of the equation was substantial at a 95% significance level as seen in Table 3. The random effect was 1.114, which was also significant at the 95% confidence level (I-95%CI, U-95%CI) = (0.0002, 3.168).

Furthermore, this is also supported by the fact that ICC (Intraclass correlation coefficient) = 0.1205 is not zero in formula (7). Because  $\vartheta^2 \neq 0$ , it is  $ICC \neq 0$ . This means that there is variability between type1 and type2, so the random effect should be taken into account. Therefore, we intend to use the generalized linear mixed model (GLMM) to estimate the fixed and random effects of digital transformation in this model<sup>7</sup>.

<sup>5</sup> There are many discussions about the size of the samples, including [Westland, 2010]. Experience shows that the ratio of analyzed situations to free parameters of 10:1 is considered sufficient. In this study, there are three parameters and 46 samples. Thus, this paper satisfies the 10:1 condition.

<sup>6</sup> After estimating the structural equation using the AMOS statistical package, the latent variables were calculated as the average of the estimated coefficients, but the factor analysis provided better results.

<sup>7</sup> The use of the Markov Monte Carlo Chain (MCMC) is intended to minimize the deviation bias between discrete values given that observations are discrete. Moreover, this method is more effective for taking insufficient variables into account.

**Table 2. Unilateral Kruskal –Wallis Test**

Variables	Statistics	$\chi^2$	Degree of freedom	P-value
Productivity (Logarithmic value)		4.9101	1	0.0415*
General price level		3.612	1	0.057(.)

Significance codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 '.' 1  
 Source: compiled by the author.

$$DT_{ij} = \mu + TYPE_j + u_{ij}$$

$$TYPE_j \sim iid N(0, \vartheta^2), u_{ij} \sim iid N(0, \Phi^2),$$

where  $\mu$  mean.

ICC calculates as follows:

$$ICC = \frac{\vartheta^2}{(\vartheta^2 + \Phi^2)}$$

**The Bayesian Approach to the Linear Multi-Level Response Mixed Model**

In the linear mixed structure with the multi-level response, the residuals calculated for the various groups during stage 2, were independent of each other. Also, it is assumed that during the first and second stages, the distribution of error is normal.

Here is an example of the latent digital transformation variable (DT), which we are trying to estimate, an average value ( $\mu$ ) and variance ( $\sigma^2$ ), about which we know nothing. In the Bayesian approach, the posterior probability density function is proportional to the likelihood function multiplied by the priori probability density function according to the rules of the Bayesian approach as follows.

$$P(\mu, \sigma^2 | DT) \propto P(DT | \mu, \sigma^2) P(\mu, \sigma^2).$$

In this study, the a priori probability density function was derived using both the non informative priori probability distribution<sup>8</sup> and the inverse Wishart priori probability distribution. In the Wishart priori probability density function, the expected mean and variance were adjusted by looking at the convergence of each variable in the case of fixed and random effects. The initial values were a variance  $\sigma^2$  of 1 and expected value  $\mu = 0.002$  in the Markov Monte Carlo model. Gibbs sampling was run from about 1,000,000 to 2,000,000 times and half was discarded to eliminate auto-correlations and dependencies from the initial value. At that time, the effective sample of about 100,000 was selected and the parameter value was estimated as the average value of the effective samples.

**The Estimated Generalized Linear Mixed Model and Results**

Generalized linear mixed models were specified at each stage to analyze the effects of the digital transformation (DT) upon productivity (PRD), general price level (PRS), and economic growth (PEG). In addition, a Bayesian approach was estimated by introducing the non informative priori probability distribution and inverse Wishart priori probability function in each equation for applying the MCMC (Markov Chain Monte Carlo).

**The Effect of Digital Transformation upon Productivity**

We analyzed the effects of digital transformation upon productivity.

$$PRD_{ij} = \alpha_{0j} + \alpha_{1j}DT_{ij} + \epsilon_{ij}, \epsilon_{ij} \sim iid N(0, \sigma^2),$$

$$\alpha_{0j} = \alpha_0 + W_{0j}, W_{0j} \sim iid N(0, \vartheta_0^2),$$

$$\alpha_{1j} = \alpha_1 + W_{1j}, W_{1j} \sim iid N(0, \vartheta_1^2),$$

$$PRD_{ij} = \alpha_0 + \alpha_1DT_{ij} + W_{0j} + W_{1j}DT_{ij} + \epsilon_{ij}$$

$$\epsilon_{ij} \sim iid N(0, \sigma^2)$$

We put equations (10) and (11) into equation (9), and yielded equation (12). In equation (12),  $j$  means type1 and type2, respectively. Moreover,  $i$  refers to individual experts in each type.

The first half of equation (12)  $\alpha_0 + \alpha_1DT_{ij}$  represents the fixed effect. The second half,  $W_{0j} + W_{1j}DT_{ij}$  represents the random effect. This section shows the size of the volatility of the intercept and the slope fluctuating around the  $\alpha_0, \alpha_1$  depending on type1 or type2. Residual  $\epsilon_{ij}$  refers to the total amount of variance that cannot be explained by DT. Also  $\epsilon_{ij}$  represents total variability within the type.  $W_{0j}$  represents the variability of the intercept due to differences between types, and  $W_{1j}$  represents the variability of the slope due to differences between types.

There are three probability variables  $\epsilon_{ij}, W_{0j}, W_{1j}$  in equation (12). Thus, there were two parameters and three probability variables to be estimated from the above model. That was  $\alpha_0, \alpha_1, W_{0j}, W_{1j}, \epsilon_{ij}$ .

**Table 3. Location Effect: logDT~1**

Intercept	Statistics	Post. mean	I-95% CI	u-95% CI	pMCMC
Intercept		1.129	0.216	2.066	0.038*

Significance codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 '.' 1  
 Source: compiled by the author.

<sup>8</sup> In general, non informative priori probability distribution means a flat distribution function, but in this study the expected average value and variance are equal to zero.

Table 4. The Effect of Digital Transformation upon Productivity

Effect	Statistics	Estimated parameter (probability variable)	Estimated value <sup>1</sup>	Credit Set (I-95%, U-95%)	P-value
Fixed effect		$\alpha_0$	1.9334	-0.67307, 4.66381	0.0738
		$\alpha_1$	0.0882	-0.24370, 0.41349	0.5886
Random effect		$W_{0j}$	292.2	0.01188, 19.11	
		$W_{1j}$	0.0603	0.01605, 0.119	
Variance of residual		$\epsilon_{ij}$	0.1043	0.04851, 0.1705	
DIC			39.49293		

<sup>1</sup> Where estimates refer to the average value and variance of precise estimates of the effective sample from the marginal probability density function.  
 Significance codes: 0 ‘\*\*\*’ 0.001 ‘\*\*’ 0.01 ‘\*’ 0.05 ‘.’ 0.1 ‘.’ 1  
 DIC — Deviance information criterion [Hadfield, 2010].  $DIC = 2D - D(\hat{\theta})$ , где  $D = -2\log(\text{Prob}(y | \hat{\theta}))$ ,  $\hat{\theta}$  is a set of parameters used in the model.  
 Source: compiled by the author.

These estimated values are summarized in Table 4. To estimate the expression (12), Gibbs sampling was repeated 2,000,000 times under the condition of the inverse Wishart probability density function. The results of Table 4 are estimated from the marginal posterior probability density function, which obtained 100,000 effective samples out of 1,000,000 left behind.

First, there was a positive relationship between digital transformation and productivity at a level of 0.0882 (fixed effect), but it was not significant. The random effect was significant and estimated at 0.0603. The total fixed effect was 2.0216. Therefore, the total effect of digital transformation upon productivity was positive and cyclical, but its statistical significance was weak.

Second, at the initial level of DT, the total random effect  $W_{0j} + W_{1j} = 292.2603$ . This value refers to the effect of DT upon PRD due to differences between groups.

Finally, the estimate of variance within the group was 0.1043. There was variability of each type was estimated at 292.2 for the intercept and 0.0603 for the slope. The dispersion in the difference between type1 and type2 should be considered significant because the dispersion figure between the groups was higher than that within groups.

**The Effect of Digital Transformation upon the General Price Level**

To analyze the effect of digital transformation upon the general price level, we created an equation system consisting of (13), (14), (15), (16), and (17). This equation system yielded random effects for all intercepts and slopes of DT and PRD by type1 and type2.

$$PR_{S_{ij}} = \beta_{0j} + \beta_{1j}DT_{ij} + \beta_{2j}PRD_{ij} + \epsilon_{ij}, \epsilon_{ij} \sim iid N(0, \rho^2), \quad (13)$$

$$\beta_{0j} = \beta_0 + U_{0j}, \quad U_{0j} \sim iid N(0, \tau_0^2), \quad (14)$$

$$\beta_{1j} = \beta_1 + U_{1j}, \quad U_{1j} \sim iid N(0, \tau_1^2), \quad (15)$$

$$\beta_{2j} = \beta_2 + U_{2j}, \quad U_{2j} \sim iid N(0, \tau_2^2), \quad (16)$$

$$PR_{S_{ij}} = \beta_0 + \beta_1DT_{ij} + \beta_2PRD_{ij} + U_{0j} + U_{1j}DT_{ij} + U_{2j}PRD_{ij} + \epsilon_{ij}, \epsilon_{ij} \sim iid N(0, \rho^2). \quad (17)$$

Equations (14), (15), and (16) were put into equation (13), and then one obtains equation (17).

The first half in the equation (17)  $\beta_{0j} + \beta_{1j}DT_{ij} + \beta_{2j}PRD_{ij}$  describes the fixed effect and the second half  $U_{0j} + U_{1j}DT_{ij} + U_{2j}PRD_{ij}$  represents the random effect. Residual  $\epsilon_{ij}$  refers to the amount of variance that cannot be explained by DT and PRD. There are four probability variables  $\epsilon_{ij}, U_{0j}, U_{1j}, U_{2j}$  in equation (17). Thus, there are three parameters and four probability variables to be estimated from the above model that were summarized in Table 5.

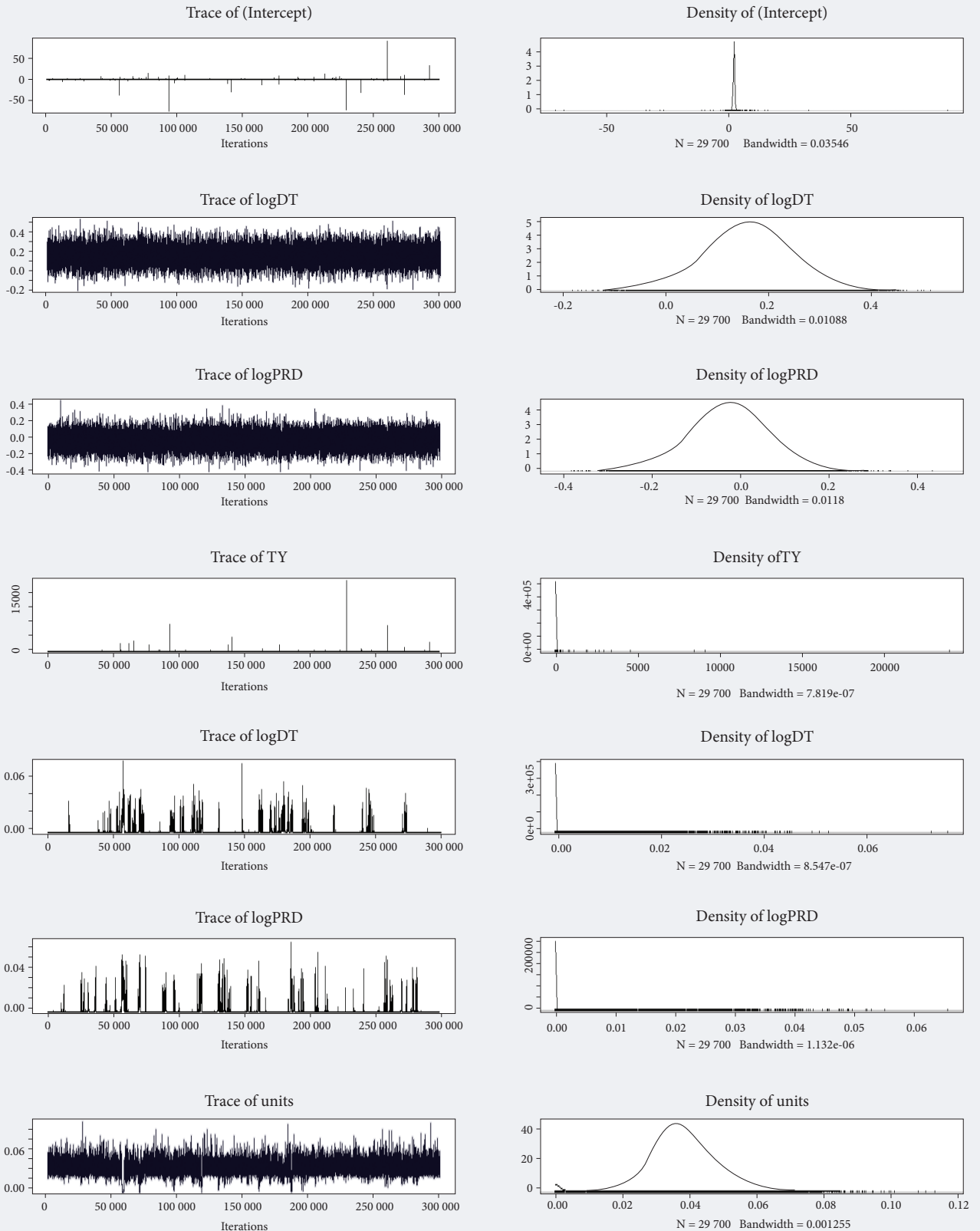
Equation (17) was estimated using the non informative priori probability density function and the inverse Wishart priori probability density function, respectively. When comparing the two models, the DIC value of the inverse Wishart priori model (-57.35371) is smaller than the non informative priori model (-18.47206). Therefore, the inverse Wishart priori model is superior to the non-informative priori model. Moreover, it is not possible to use the non-informative priori distribution because all the variables are unstable and not converging with the random effect as illustrated in Figure 3. On the other hand, each variable of the random effect derived under the inverse Wishart distribution is converging in Figure 4.<sup>9</sup> Therefore, the effect of digital transformation upon the general price level is to be analyzed with estimates obtained on the basis of the inverse Wishart probability distribution.

First, the effect of digital transformation upon the general price level in fixed effects was  $\beta_1=0.1609$  at

<sup>9</sup> All variables, regardless of the form of all priori information functions, were converged in the fixed effect.

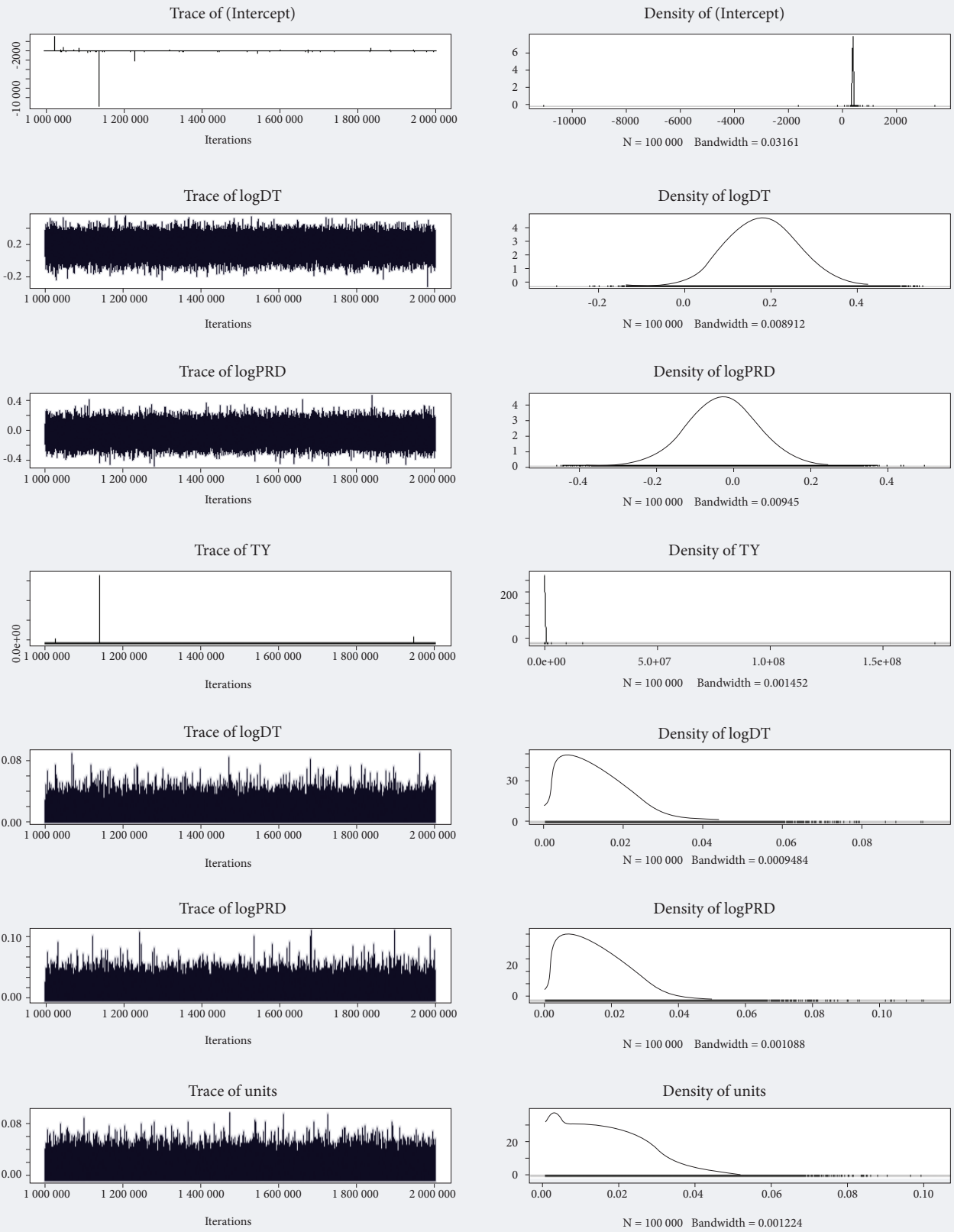


**Figure 3. Marginal Posterior Probability Distribution Function with the Use of the Non-Informative Priori Probability Distribution Function in Equation (17)**



Source: compiled by the author.

**Figure 4. Marginal Posterior Probability Distribution Function under Inverse Wishart Priori Probability Distribution Function in Equation (17)**



Source: compiled by the author.

Table 5. Effect of DT upon General Price Level

Item		Non-Informative Priori Distribution			Inverse Non-Informative Priori Distribution		
Statistics	Estimate parameter (probability variable)	Average	Credit set (1-95%, U-95%)	P-value	Average	Credit set (1-95%, U-95%)	P-value
Fixed effect	$\beta_0$	2.3659	1.841143, 2.903513	0.00162 **	2.3312	1.732990, 3.108864	0.0085 **
	$\beta_1$	0.1652	0.001229, 0.321681	0.0421 *	0.1609	-0.005971, 0.326778	0.0571
	$\beta_2$	-0.0232	-0.196255, 0.150911	0.7878	-0.0438	-0.216958, 0.136778	0.6184
Random effect	$U_{0j}$	2.3750	1.784e-17, 0.009582		2018	1.149e-05, 0.5329	
	$U_{1j}$	0.0011	7.634e-17, 0.007833		0.0133	0.000652, 0.03041	
	$U_{2j}$	0.0012	8.439e-17, 0.007718		0.0159	0.0009982, 0.03512	
Variance of residual	$\varepsilon_{ij}$	0.0385	0.0184, 0.06123		0.0166	0.0002196, 0.03767	
DIC		-18.47206			-57.35371		

Significance codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
 DIC — Deviance information criterion  
 Source: compiled by the author.

marginally statistical significance. Digital transformation significantly raised prices rather than lowered them. This means that digital transformation did not lead to productivity gains and a fall in prices, but increased costs. A similar effect has also been shown to raise prices in random effects that significantly reflect ( $U_{1j}=0.0133$ ). Thus, digital transformation significantly increases prices for both fixed and random effects.

Second, the effect of productivity on prices is different for fixed effects and random effects. Fixed effects prompt an insignificant drop in prices, while random effects drive prices up (at a confidence level of 95%). The effect of productivity upon the price level was not clear.

Finally, the estimate of variance within the types is significant at 0.0166 at a confidence level of 95%. This is less than the dispersion between type1 and type2. This means that although the variation of general price level comes from within the group, one should also consider the variability resulting from the differences between type1 and type2. All the above estimated values are formed within a confidence level of 95%.

### The Effect of Digital Transformation upon Economic Growth

Let us analyze the effect of digital transformation upon economic growth. To reflect the difference between type1 and type2, an equation reflecting random effects upon the intercept and the slope of DT and PRD was created as follows.

$$PEG_{ij} = \gamma_{0j} + \gamma_{1j}DT_{ij} + \gamma_{2j}PRD_{ij} + v_{ij}, v_{ij} \sim iid N(0, \phi^2), \quad (18)$$

$$\gamma_{0j} = \gamma_0 + V_{0j}, V_{0j} \sim iid N(0, \phi_0^2), \quad (19)$$

$$\gamma_{1j} = \gamma_1 + V_{1j}, V_{1j} \sim iid N(0, \phi_1^2), \quad (20)$$

$$\gamma_{2j} = \gamma_2 + V_{2j}, V_{2j} \sim iid N(0, \phi_2^2), \quad (21)$$

Let us put equations (19), (20), and (21) into equation (18), and then we can obtain equation (22).

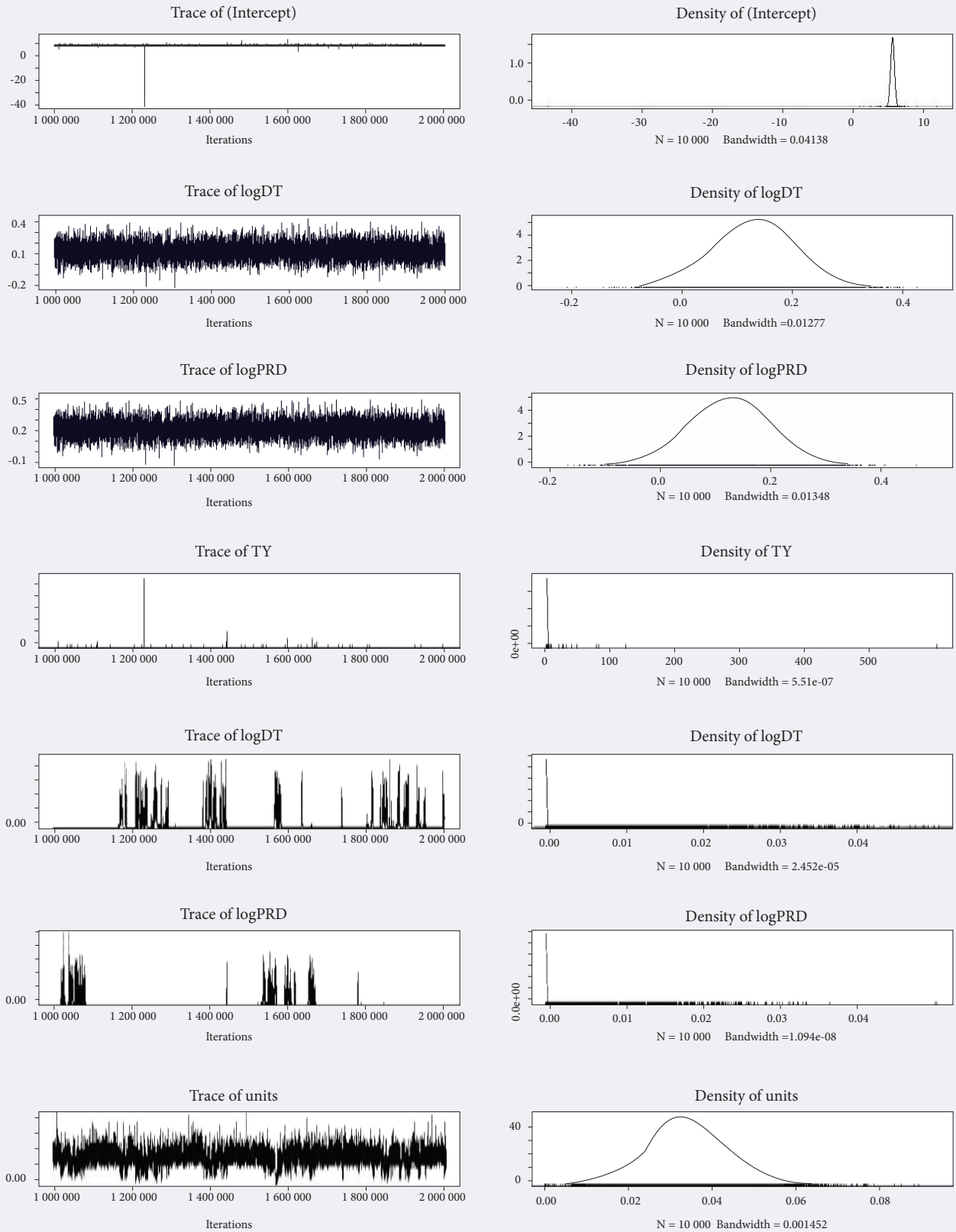
$$PEG_{ij} = \gamma_0 + \gamma_1DT_{ij} + \gamma_2PRD_{ij} + V_{0j} + V_{1j}DT_{ij} + V_{2j}PRD_{ij} + v_{ij}, v_{ij} \sim iid N(0, \phi^2). \quad (22)$$

The first half of the equation (22)  $\gamma_0 + \gamma_1DT_{ij} + \gamma_2PRD_{ij}$  represents the fixed effect, and the second half  $V_{0j} + V_{1j}DT_{ij} + V_{2j}PRD_{ij}$  describes the random effect. Residual  $v_{ij}$  refers to the total amount of variance that cannot be explained by DT and PRD. There were four probability variables  $v_{ij}, V_{0j}, V_{1j}$  and  $V_{2j}$  in the equation (22). Thus, there were three parameters and four probability variables to be estimated from equation (22). The results were summarized in Table 6.

Equation (22) was estimated using the non-informative priori probability density function and inverse Wishart priori probability density function. When comparing the two models, the DIC value of the inverse Wishart priori model (-94.69512) is smaller than the non-informative model (-23.1925) in Table 7. Therefore, the inverse Wishart model is superior to the non-informative priori model. Moreover, it is not possible to use the non-informative priori distribution each variable is unstable and does not converge with the random effect in Figure 5.<sup>10</sup> On the other hand, each variable of the random effect derived under the inverse Wishart priori distribution converges in Figure 6. Therefore, the effect

<sup>10</sup> All variables, regardless of the form of all prior information functions, converge with the fixed effect.

Figure 5. Marginal Posterior Probability Distribution Function with the Use of a Non-Informative Priori Probability Distribution Function in Equation (22)



Source: compiled by the author.

of digital transformation upon economic growth should be analyzed with estimates obtained on the basis of the inverse Wishart non-informative probability distribution.

First, digital transformation has a positive effect upon economic growth with fixed effects ( $\gamma_1 = 0.1379$ ) at a marginally significant level. For random effects, there was a positive relationship ( $V_{ij} = 0.0176$ ) at a 95% confidence level. Digital transformation demonstrates positive effects upon economic growth both in terms of fixed and random effects. This means that digital transformation can play a powerful role in driving economic growth in Russia.

Second, it can also be inferred that productivity has a marginally significant impact upon economic growth both in terms of the fixed effect ( $\gamma_2 = 0.1654$ ) and random effect ( $V_{ij} = 0.0150$ ) with a 95% confidence level. It can be thought that digital transformation has a positive effect upon economic growth via two channels. One manifests itself directly through technological advances and the other does so indirectly through productivity improvements.

Third, the estimate of the variance within the group is 0.0071 and the variation between groups is 699.4 for the intercept at a 95% confidence level. This means that the differences between the groups also have a significant effect.

### Analysis of the Random Interaction Effect and Digital Transformation

The random interaction effect of digital transformation upon the general price level and economic growth is analyzed using a variance function. In order to analyze type1 and type2 by DT or PRD inter-

action, we use variance function as illustrated below in (23), (24)

$$V_{DT} = \begin{bmatrix} V_{1,1} & V_{1,2} \\ V_{2,1} & V_{2,2} \end{bmatrix} = \begin{bmatrix} \sigma_{type1}^2 & \sigma_{type1, type2} \\ \sigma_{type2, type1} & \sigma_{type2}^2 \end{bmatrix} \quad (23)$$

$$V_{PRD} = \begin{bmatrix} V_{1,1} & V_{1,2} \\ V_{2,1} & V_{2,2} \end{bmatrix} = \begin{bmatrix} \sigma_{type1}^2 & \sigma_{type1, type2} \\ \sigma_{type2, type1} & \sigma_{type2}^2 \end{bmatrix} \quad (24)$$

We assume that the different types in DT or PRD are independent, so variance function (23-1), (24-1),  $V_{1,2} = V_{2,1}$  is equal to zero, we could see no relationship between type1 and type2. On the basis of this, an attempt was made to evaluate the dispersion caused by the interactions of DT and PRD within type1 and type2, respectively [Hadfield, 2019].

$$V_{DT} = \begin{bmatrix} V_{1,1} & V_{1,2} \\ V_{2,1} & V_{2,2} \end{bmatrix} = \begin{bmatrix} \sigma_{type1}^2 & 0 \\ 0 & \sigma_{type2}^2 \end{bmatrix} \quad (23-1)$$

$$V_{PRD} = \begin{bmatrix} V_{1,1} & V_{1,2} \\ V_{2,1} & V_{2,2} \end{bmatrix} = \begin{bmatrix} \sigma_{type1}^2 & 0 \\ 0 & \sigma_{type2}^2 \end{bmatrix} \quad (24-1)$$

If the variance function is introduced in the random effect, the priori probability distribution should be set up differently than it has been in the analysis so far. This is because the variance function is obtained using a matrix, not by a scalar value. If the matrix in Equation (23-1) and (24-1) is reflected in the inverse Wishart priori probability distribution, then the posterior probability density function will be changed as the likelihood function is changed.

### The Random Interaction Effect of Digital Transformation upon the General Price Level

The interaction effect of digital transformation in fact is the effect of a whole range of factors, so we

Table 6. The Effect of DT upon Economic Growth

Item		Non-Informative Priori Distribution			Inverse Wishart Priori Distribution		
Statistics Effects	Estimated parameter	Estimated value	Confidence level (I-95%, U-95%)	P- value	Estimated value	Confidence level (I-95%, U-95%)	P- value
Fixed effect	$\gamma_0$	2.7668	2.298004, 3.255209	0.001***	2.6861	1.352031, 4.119781	0.0249*
	$\gamma_1$	0.1388	-0.012589, 0.290072	0.0742(.)	0.1379	0.014989, 0.291980	0.0766(.)
	$\gamma_2$	0.1428	-0.007986, 0.309799	0.0780(.)	0.1654	0.004368, 0.324948	0.0434*
Random effect	$V_{0j}$	0.1201	2.295e-17, 0.006076		699.4	0.0009326, 5.569	
	$V_{1j}$	0.0022	1.206e-16, 0.01666		0.0176	0.003383, 0.03356	
	$V_{2j}$	0.0009	9.235e-17, 0.006303		0.0150	0.002911, 0.02884	
Variance of residual	$v_{ij}$	0.0322	0.01144, 0.05125		0.0071	0.000173, 0.02024	
DIC		-23.1925			-94.69512		

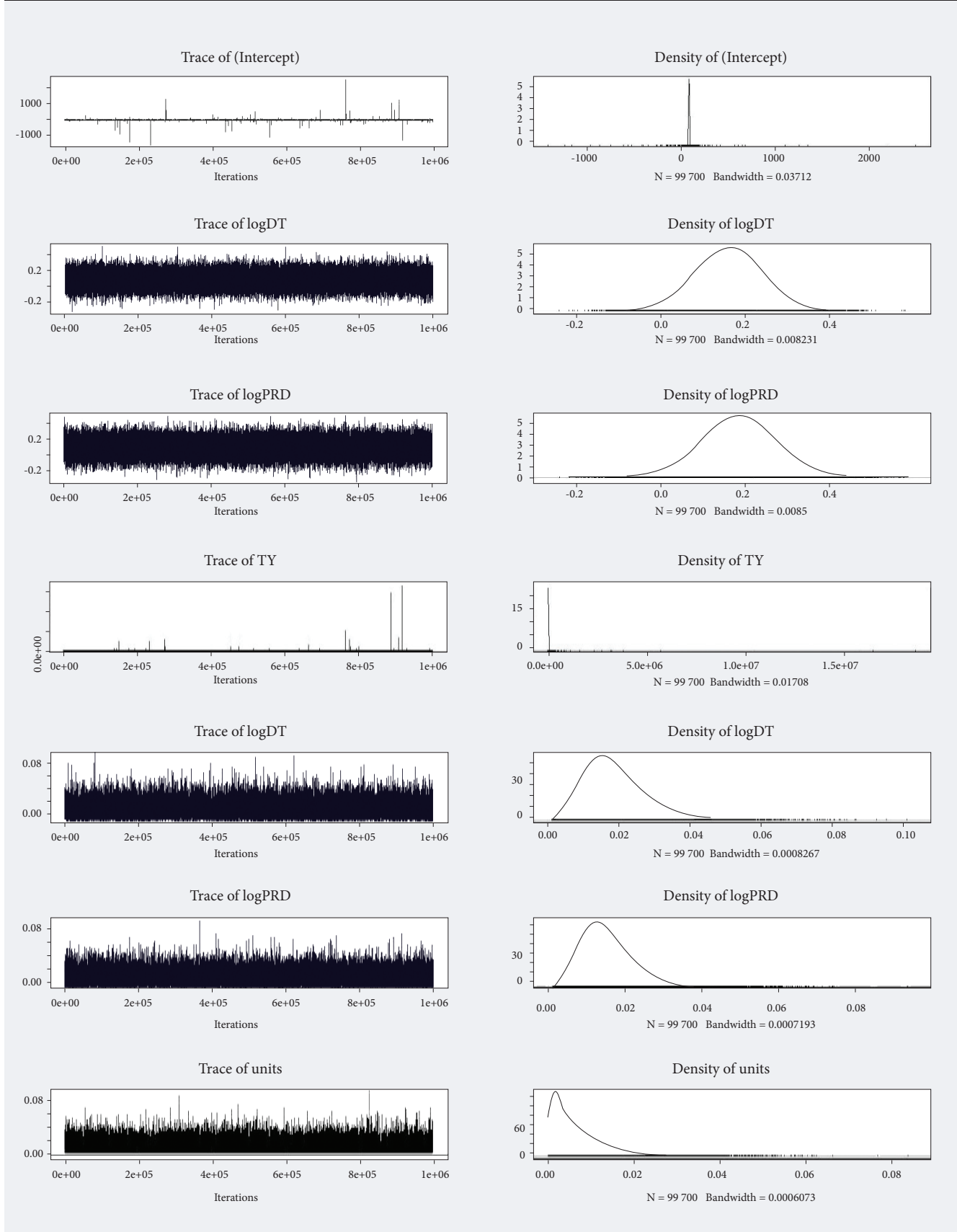
Significance codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

DIC — Deviance information criterion

Source: compiled by the author.



**Figure 6. Marginal Posterior Probability Distribution Function Using the Inverse Wishart Prior Probability Distribution Function in Equation (22)**



Source: compiled by the author.

Table 7. The Interaction Effect of DT upon General Price Level

Effect	Statistics	Estimated parameter	Post-mean	Credit Set (I-95%, U-95%)	P-value
Fixed effect		$\beta_0$	2.3022	1.63339, 2.97441	<1e-05***
		$\beta_1$	0.2094	0.01253, 0.41527	0.0431*
		$\beta_2$	-0.0552	-0.25718, 0.14320	0.5760
Random interaction effect		TY:TY.logDT	0.0070	0.003159, 0.01154	
		TY:TY.logPRD	0.0064	0.002962, 0.01076	
Variance of residual		Variance	0.0041	0.0001579, 0.0123	
Fitting degree of model		DIC	-113.17		

Significance codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
 DIC — Deviance information criterion  
 Source: compiled by the author.

looked at the impact of logDT and logPRD in relation to general price level in the type1 and type2 groups. The variance function (23) describes the effect of the respective logDT (in type1 and type2) in the random effect. Another function (24) also describes the impact of the corresponding logPRD (also for type1 and type2) in the random effect. The results are summarized in Table 7.

In Table 7, DIC = -113.17 is so low that it is clear that it confirms the high degree of conformity of the model. In fact, in Figure 7, time traces representing MCMC (Markov Chain Monte Carlo) of each variable are well scattered up and down. The marginal posterior probability density function, which is drawn with effective samples is symmetrically shaped well

In relation to the fixed effect (0.2094), it was shown that digital transformation leads to a substantial rise in prices. Productivity growth (-0.0552) lowers prices but is not significant. This is a similar result to the previous analysis without considering the random interaction effect. Thus, one may postulate that digital transformation will lead to a rise in prices whether or not the random interaction effect is considered.

The variability (0.0070) of logDT in the type1 and type2 groups has been shown to significantly increase the price level. Productivity fluctuation (0.0064) has also been shown to significantly increase prices. There is little difference between the two values, but the variability caused by the interaction of logDT is greater than the variability caused by the interaction of logPRD.

Therefore, this suggests that in Russia, although in the early stages, digital transformation is linked to growing costs and prices rather than to investment and productivity improvements. All estimates except productivity have a 95 % confidence level.

Figure 7 shows the MCMC of the intercept, logDT, and logPRD respectively. The left-hand figure shows the 1,000,000 time traces of the parameters. The first 500,000 times are excluded to remove the influences of the initial value of the inverse Wishart

probability distribution. The right-hand side shows the marginal posterior probability function of the estimated parameters from the effective samples. Estimates of each variable were derived from the stationary state of the picture on the right. The intercept fluctuates around about 2.3 and the scatter is not large, indicating that the estimated model is stable. The marginal posterior probability function of logDT and logPRD is also symmetrical to the left and right, so it can be seen to show an almost normal distribution. The logDT and logPRD also fluctuate around 0.2 or -0.05. It is symmetrical to the left and right, showing a similar approach to normal distribution.

In the graph, the random interaction effects of logDT and logPRD are also reliably converging. LogDT is centered around 0.0070 and logPRD shows a normal distribution of symmetry from left to right at 0.0064. The variance demonstrates some instability.

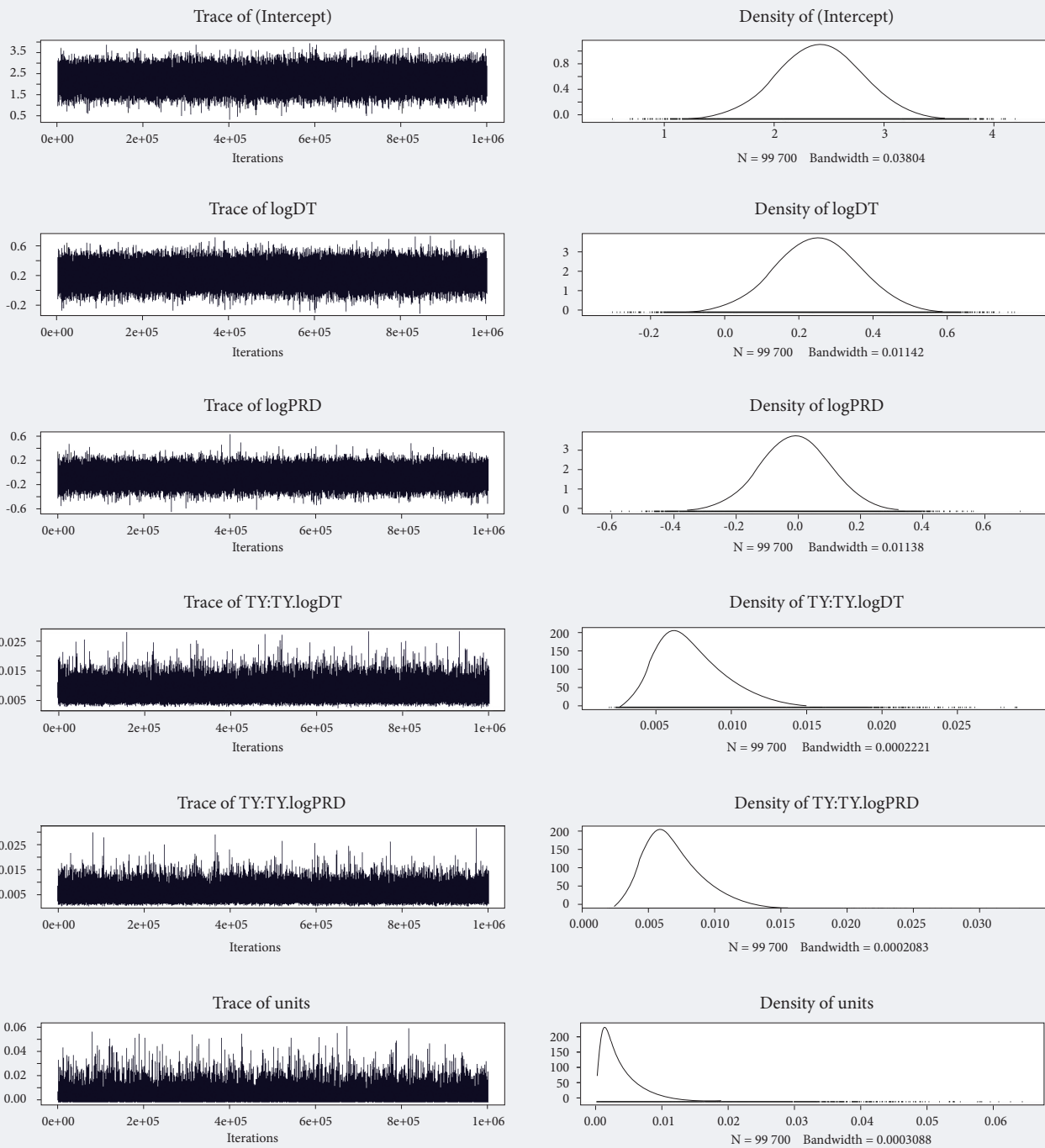
### ***The Random Interaction Effect of Digital Transformation upon Economic Growth***

The variance structure (23), (24) was substituted for the random effect equation (22) to see the random effect's interaction with economic growth.

In Table 8, the DIC is -31.07074 very low. Therefore, we can see that the model has a high level of conformity. In Figure 8, we can also see that all variables with fixed effects are converging. However, the marginal posterior probability function of both logDT and logPRD's random interaction effects skews to the left. This is due to the influences of the initial expected value. As we increase the number of repetitions, it is expected that we will approach a normal distribution.

The effect of logDT and logPRD upon economic growth for fixed effects is 0.1528 and 0.1355, respectively, with a marginally significant positive effect. The variability of logDT in the random interaction effect is 0.0015, which is greater than the variability (0.0010) of logPRD. Both values were significant at the 95% confidence level. In the equation, the vari-

Figure 7. Marginal Posterior Distribution of the Random Interaction Effect on logPRS



Source: compiled by the author.

ance estimate of residuals is 0.0207 and the function is steadily converging while the distribution of the marginal posterior probability is almost normal.

### Conclusions

On the basis of the Bayesian approach to the analysis of a cross-section of latent variables (data for 2018) and the rational expectation theory, this paper draws the following conclusions.

First, the fixed effect of digital transformation upon productivity was not significant. However, in terms of the random effect, digital transformation had a significant positive impact. It is not easy to say that digital transformation has a positive effect upon productivity with a significant random effect but no fixed effect.

Second, both in terms of fixed and random effects, digital transformation has raised prices regardless of the form of the a priori probability distribution

function. Digital transformation raises prices because its impact upon productivity remains unclear.

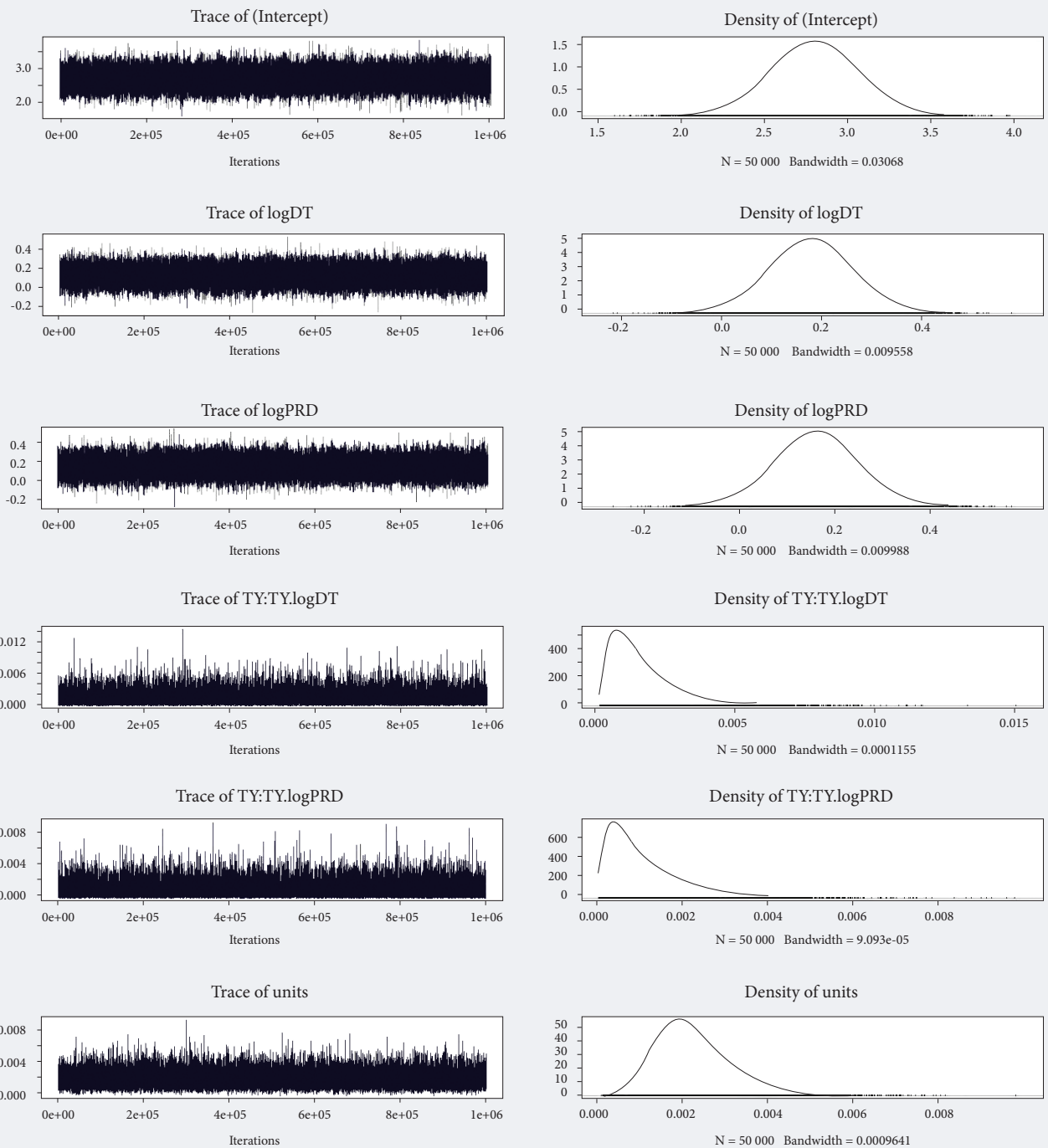
Third, when evaluating the effect of random interaction (with account of the variance function) fluctuations in the evaluation of this impact within groups was statistically meaningful, but generally digital transformation facilitates the increase of prices, These three results suggest that Russia needs to implement an innovation policy when pursuing digital

transformation to stabilize prices through productivity improvement in the future.

Fourth, because the evaluations made by the pivotal and non-pivotal groups affected the variances of the general price level and economic growth, the differences between these groups should be considered

Fifth, digital transformation and productivity have demonstrated a statistically and consistently significant positive effect upon economic growth in terms

Figure 8. Marginal Posterior Distribution of Random Interaction Effect on logPEG



Source: compiled by the author.

**Table 8. Interaction Effect of DT upon Economic Growth**

Effect	Statistics	Estimated parameter	Post-mean	Credit Set (l-95%, U-95%)	P-value
Fixed effect		$\gamma_0$	2.7641	2.264627, 3.267249	2e-05***
		$\gamma_1$	0.1528	0.004667, 0.307963	0.0561
		$\gamma_2$	0.1355	0.024912, 0.300192	0.0999
Random interaction effect		TY:TY.logDT	0.0015	0.000115, 0.003585	
		TY:TY.logPRD	0.0010	5.066e-05, 0.002675	
Variance of residual		Variance	0.0207	0.006324, 0.0373	
Fitting degree of model		DIC		-31.07074	

Significance codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
 DIC — Deviance information criterion  
 Source: compiled by the author.

of both fixed and random effects. These results occurred regardless of the type of priori distribution, but when the inverse Wishart priori distribution was used, it was more stable as variables were converging, unlike the non-informative priori distribution. Sixth, the random effect of digital transformation and productivity in relation to economic growth turned out to be substantial during the analysis of both groups. The random interaction effect of digital transformation and economic growth was more significant than that of the random interaction with productivity. One might conclude that the development of digital technologies directly impact economic growth. In addition, according to the respondents, digital transformation is thought to have a positive impact upon economic growth indirectly,

through the improvement of productivity. This is clear evidence that in Russia the digital transformation is recognized as a technology shock affecting economic growth.

Therefore, in Russia in 2018, digital transformation has played a role in terms of technological progress that attracts economic growth rather than economic stability.

This paper has certain limitations. During the analysis with the use of the multi-level linear model and the Bayesian approach to variables of digital transformation, productivity, general price level, and economic growth were evaluated on the basis of measured variables, and not on actual data. In the future, these results must be empirically tested despite the difficulty of obtaining relevant real data.

**References**

Akerlof G.A. (1984) Gift Exchange and Efficiency-Wage Theory: Four Views. *The American Economic Review*, vol. 74, no 2, pp. 79–83.

Anderson T.W., Rubin H. (1956) Statistical Inference in Factor Analysis. *Proceedings of the Third Berkeley Symposium on Mathematical Statistics and Probability*, vol. 5. Berkeley, CA: University of California Press, pp. 111–150.

Ball L., Romer D. (1990) Real Rigidities and the Non-Neutrality of Money. *The Review of Economic Studies*, vol. 57, no 2, pp. 183–203.

Bartholomew D., Knott M., Moustaki I. (2011) *Latent Variable Models and Factor Analysis: A Unified Approach* (3<sup>rd</sup> ed.), Hoboken, NJ: John Wiley & Sons, pp. 157–189.

Caballé J., Santos M.S. (1993) On Endogenous Growth with Physical and Human Capital. *Journal of Political Economy*, vol. 101, no 6, pp. 1042–1067.

Davis J.M.V., Guryan J., Hallberg K., Ludwig J. (2017) *The Economics of Scale-Up*. NBER Working Paper no 23925. Cambridge, MA: NBER.

Draco M., Sadun R., van Reenen J. (2015) *Productivity and ICT: A Review of the Evidence*, CEP Discussion Paper 749, London: Center for Economic Performance.

Friedman M. (2017) Quantity Theory of Money. *The New Palgrave Dictionary of Economics*, pp. 1–31. Available at: [https://miltonfriedman.hoover.org/friedman\\_images/Collections/2016c21/Palgrave\\_1987\\_c.pdf](https://miltonfriedman.hoover.org/friedman_images/Collections/2016c21/Palgrave_1987_c.pdf), accessed 24.11.2019.

Goldfarb A., Greestein S.M., Tucker C.E. (eds.) (2015) *Economic Analysis of Digital Economy*, Chicago: University of Chicago Press.

Hadfield J. (2010) MCMC Methods for Multi-Response Generalized Linear Mixed Models: The MCMCglmm R Package. *Journal of Statistical Software*, vol. 33, no 2, pp. 1–22. Available at: <https://doi.org/10.18637/jss.v033.i02>, accessed 15.10.2019.



- Hadfield J. (2019) *MCMCglmm Course Notes*. Available at: <https://cran.r-project.org/web/packages/MCMCglmm/vignettes/CourseNotes.pdf>, accessed 15.10.2019.
- Howitt P. (1999) Steady Endogenous Growth with Population and R&D Inputs Growing. *Journal of Political Economy*, vol. 107, no 4, pp. 715–730.
- Jones C.I. (1995) Time Series Tests of Endogenous Growth Models. *The Quarterly Journal of Economics*, vol. 110, no 2, pp. 495–525.
- Joreskog K.G. (1990) New Developments in LISREL Analysis of ordinal variables using poly-choric correlations and weighted least squares. *Quality and Quantity*, vol. 24, pp. 387–404.
- Lawley D.N., Maxwell A.E. (1962) Factor Analysis as a Statistical Method. *Journal of the Royal Statistical Society. Series D (The Statistician)*, vol. 12, no 3, pp. 209–229.
- Lucas R.E. (1972) Expectations and the neutrality of money. *Journal of Economic Theory*, vol. 4, no 2, pp. 103–124.
- Moulin H. (1986) Characterizations of the Pivotal Mechanism. *Journal of Public Economics*, vol. 31, no 1, pp. 53–78.
- Solow R.M. (1987) “We’d Better Watch out”. Review of S.S. Cohen and J. Zysman, *Manufacturing Matters: The Myth of the Post-Industrial Economy*. New York Times, 12.07.1987. Available at: [https://pdfs.semanticscholar.org/cef1/49b3dbdaa85f74b114c2c7832982f23bcbf0.pdf?\\_ga=2.192560554.1655282957.1574608201-410801543.1574608201](https://pdfs.semanticscholar.org/cef1/49b3dbdaa85f74b114c2c7832982f23bcbf0.pdf?_ga=2.192560554.1655282957.1574608201-410801543.1574608201), accessed 26.10.2019.
- Triplitt J.E. (1999) The Solow Productivity Paradox: What do computers do to productivity? *The Canadian Journal of Economics*, vol. 32, no 2, pp. 310–334.
- Westland J.C. (2010) Lower Bounds on Sample Size in Structural Equation Modelling. *Electronic Commerce Research and Applications*, vol. 9, no 6, pp. 476–487.